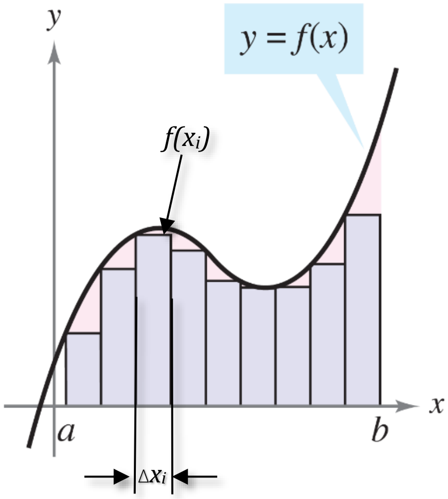
定积分 Definite Integration(p273)

Sigma Notation: The sum of n terms $a_1, a_2, a_3, \dots a_n$ is written as

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$



用面积定义定积分 The definite integral as the area of a region.(p274)

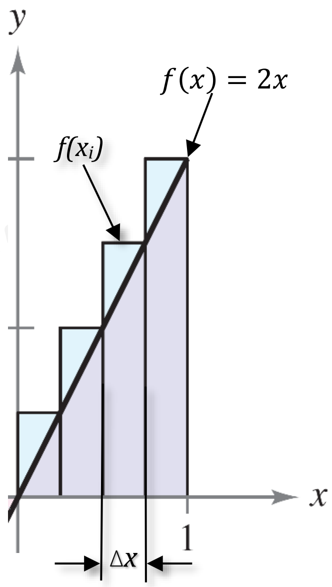
If $f(x)$ is continuous and nonnegative on the closed interval $[a, b]$, then the area is:

$$Area = \int_a^b f(x)dx$$

用极限定义定积分 If $f(x)$ is defined on the closed interval $[a,b]$ and the limit of

$$\lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

is called the **definite integral** of $f(x)$ from a to b.



Example: Evaluate the definite integral $\int_0^1 2x\,dx$

Solution 将$[a,b]$区间分为 n 份，每份是一个小长方形。每一个小长方形:

宽：$\Delta x = \frac{b-a}{n} = \frac{1}{n}$

高：$f(x_i) = a + 2i\Delta x = 2i/n$

$$S = \int_0^1 2x\,dx = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{\Delta x \to 0} \sum_{i=1}^{n} \left(\frac{2i}{n} \cdot \frac{1}{n}\right)$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2i}{n^2} = \lim_{n \to \infty} \frac{2(1 + 2 + \cdots + n)}{n^2} = \lim_{n \to \infty} \frac{2\left[\frac{1}{2}(1+n)n\right]}{n^2}$$

$$= \lim_{n \to \infty} \frac{n + n^2}{n^2} = 0 + 1 = 1$$

定积分的性质(p276)

$$\int_a^a f(x)\,dx = 0 \qquad \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

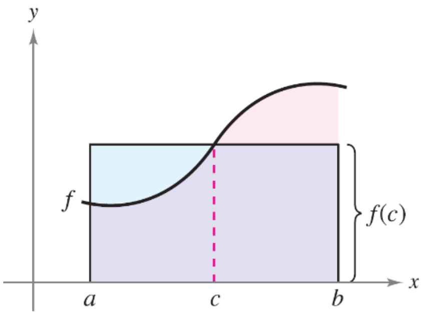
$$\int_a^b f(x)\,dx \geq 0 \qquad \text{if } f(x) \text{ is integrable and no negative on } [a,b]$$

$$\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx \qquad \text{if } f(x) \geq g(x) on\ [a,b]$$

积分中值定理 Mean value theorem for integrals(285)

If $f(x)$ is continuous on [a,b], then there exists a number c in [a,b] such that

$$\int_a^b f(x)\,dx = f(c)(b-a)$$



Mean value rectangle:

$$f(c)(b-a) = \int_a^b f(x)\,dx$$

牛顿-莱布尼兹公式(p282)the fundamental theorem of calculus

If $f(x)$ is continuous on [a, b] and $F(x)$ is an antiderivative of $f(x)$ on [a, b], then

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \qquad \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$$

Proof: **【定积分部分】**

如果 $f(t)$连续，$\int_a^b f(t)\,dt$是函数 $f(t)$在[a,b]上的定积分。如果积分上限 b 从固定量改为变量 $t(b \to t)$，这将是一个关于积分上限t的函数。定义一个函数 $F(t) = \int_a^t f(t)\,dt$（变上限定积分-见 p288）。

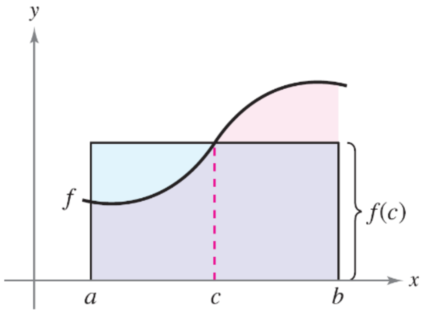
注意：在这里$F(t)$仅仅是用定积分定义的$f(t)$的面积的函数，不是$f(t)$的原函数。我们下一步的工作是证明$F(t)$就是$f(t)$的原函数。

【函数求导部分】

$$F'(x) = \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\int_a^{x+\Delta x} f(t)dt - \int_a^x f(t)dt}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \int_x^{x+\Delta x} f(t)\,dt \xrightarrow[x \le c \le x+\Delta x]{mean\ value\ theorem}$$

$$f(c)\Delta x = \int_x^{x+\Delta x} f(t)\,dt \Rightarrow F'(x) = \lim_{\Delta x \to 0} f(c) = f(x)$$



Mean value rectangle:

$$f(c)(b-a) = \int_a^b f(x)\,dx$$

具体证明过程：https://youtu.be/pWtt0AvU0KA

课本 p283 有另外一种证明方法，是用求和方法+中值定理证明。

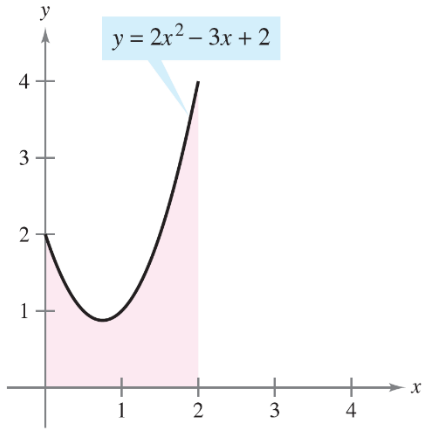
Example: Evaluate each definite integral.

$$\int_1^2 (x^2 - 3)\,dx = \left[\frac{x^3}{3} - 3x\right]_1^2 = \left(\frac{8}{3} - 6\right) - \left(\frac{1}{3} - 3\right) = -\frac{2}{3}$$

$$\int_0^{\pi/4} (sec^2 x)\,dx = \left[\tan x\right]_0^{\pi/4} = 1 - 0 = 1$$

Example: Using the fundamental theorem to find area

Find the area of the region bounded by the graph $y = 2x^2 - 3x + 2$, the x-axis, and the vertical lines $x = 0$ and $x = 2$.



The area of the region bounded by the graph of y, the x-axis, $x = 0$, and $x = 2$ is $\frac{10}{3}$.

$$\text{Area} = \int_0^2 (2x^2 - 3x + 2)\,dx \qquad \text{Integrate between } x = 0 \text{ and } x = 2.$$

$$= \left[\frac{2x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^2 \qquad \text{Find antiderivative.}$$

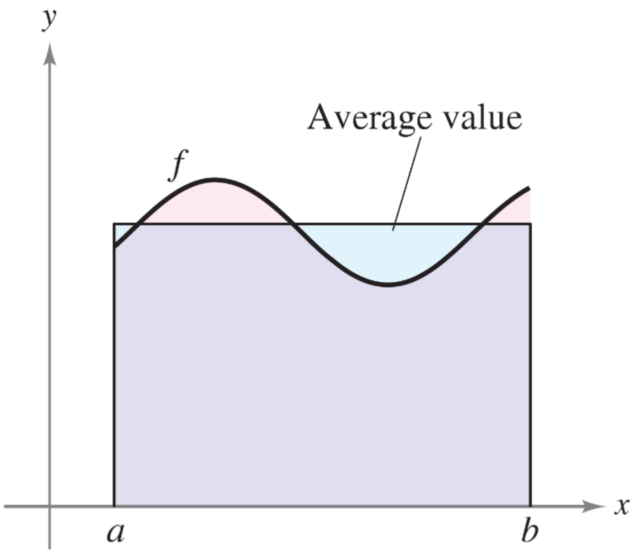
$$= \left(\frac{16}{3} - 6 + 4\right) - (0 - 0 + 0) \qquad \text{Apply Fundamental Theorem.}$$

$$= \frac{10}{3} \qquad \text{Simplify.}$$

函数的平均值 Average value of a function(p286)

If $f(x)$ is integrable on [a,b], the average value of $f(x)$ on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$



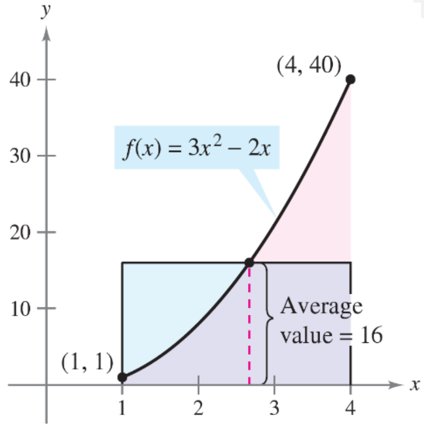
Example: Find the average value of $f(x) = 3x^2 - 2x$ on the interval [1,4]

Solution The average value is

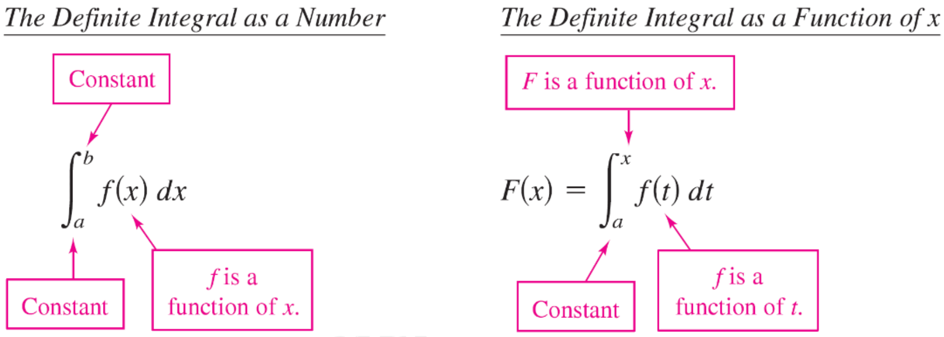
$$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{4-1}\int_1^4 (3x^2-2x)\,dx$$

$$= \frac{1}{3}[x^3 - x^2]_1^4 = \frac{1}{3}[64 - 16 - (1-1)]$$

$$= \frac{48}{3} = 16$$



变上限定积分 Definite Integral with Variable as Upper Limit(288)



<u>积分形式</u> <u>微分形式</u>

$$F(x) = \int_0^x f(t)\,dt \qquad \frac{d}{dx}\left[\int_0^x f(t)\,dt\right] = f(x)$$

Example: Evaluate the function

$$F(x) = \int_0^x \cos t\,dt \qquad as \quad x = \pi/6 \quad and \quad \pi/2$$

Solution

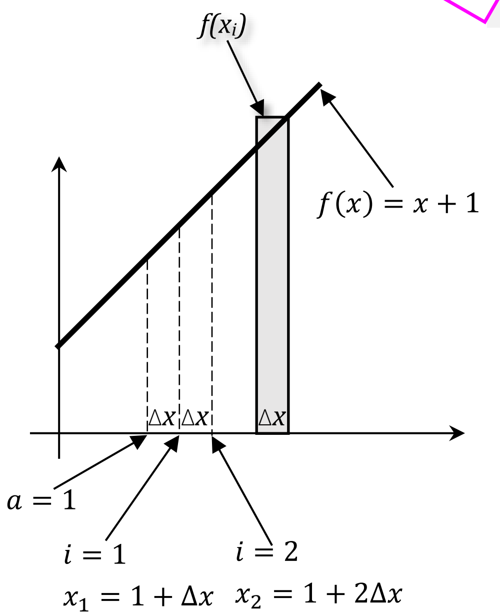
$$\int_0^x \cos t\,dt = [\sin t]_0^x = \sin x - \sin 0 = \sin x$$

$$F(\pi/6) = \sin \pi/6 = 1/2$$

$$F(\pi/2) = \sin \pi/2 = 1$$

Exercise 1: Evaluate the following definite integrals as a limit

$$\int_1^3 f(x)\,dx = \int_1^3 (x+1)\,dx$$



Solution 将[a,b]区间分为 n 份，每份是一个小长方形。每一个小长方形:

$$\Delta x = \frac{\mathrm{b}-\mathrm{a}}{\mathrm{n}} = \frac{3-1}{\mathrm{n}} = \frac{2}{n}$$

$$x_1 = a + 1\cdot\Delta x = 1 + 1\cdot\Delta x$$

$$x_2 = a + 2\cdot\Delta x = 1 + 2\cdot\Delta x$$

$$x_i = a + i\cdot\Delta x = 1 + i\cdot\Delta x$$

$$f(x_i) = x_i + 1 = (1 + i\cdot\Delta x) + 1$$

$$= 2 + i\cdot\Delta x = 2 + \mathrm{i}\frac{2}{n}$$

$$S = \int_1^3 (x+1)\,dx = \lim_{\Delta x_i\to 0}\sum_{i=1}^{n} f(x_i)\Delta x_i$$

$$= \lim_{\Delta x\to 0}\sum_{i=1}^{n}\left[\left(2 + i\cdot\frac{2}{n}\right)\cdot\frac{2}{n}\right] = \lim_{\Delta x\to 0}\sum_{i=1}^{n}\left(\frac{4}{n}+\frac{4i}{n^2}\right) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{4}{n} + \lim_{n\to\infty}\sum_{i=1}^{n}\frac{4i}{n^2}$$

$$= \lim_{n\to\infty}\left(\frac{4}{n}+\frac{4}{n}+\cdots+\frac{4}{n}\right) + \lim_{n\to\infty}\left(\frac{4\cdot 1 + 4\cdot 2 + \cdots + 4\cdot n}{n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{4n}{n}\right) + \lim_{n\to\infty}\left(\frac{\frac{(4+4n)n}{2}}{n^2}\right) = 4 + \lim_{n\to\infty}\left(\frac{2+2n^2}{n^2}\right)$$

$$= 4 + \lim_{n\to\infty}\frac{2}{n^2} + \lim_{n\to\infty} 2 = 4 + 0 + 2 = 6$$

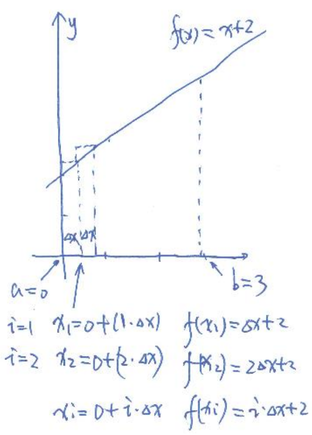
Exercise 2: Evaluate the limit

$$\lim_{\Delta x_i\to 0}\sum_{i=1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

Over the region bounded by the graphs of the equations.

$$f(x) = x + 2, y = 0, x = 0, x = 3$$

Solution $\Delta x = \frac{\mathrm{b-a}}{\mathrm{n}} = \frac{3-0}{\mathrm{n}} = \frac{3}{n}$



$$x_1 = 0 + 1\cdot\Delta x$$

$$x_2 = 0 + 2\cdot\Delta x$$

$$x_i = 0 + i\cdot\Delta x$$

$$f(x_i) = x_i + 2 = i\cdot\Delta x + 2 = i(3/n) + 2$$

$$S = \lim_{\Delta x\to 0}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(i\frac{3}{n}+2\right)\frac{3}{n}\right]$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(i\frac{3}{n}\right)\frac{3}{n}\right] + \lim_{n\to\infty}\sum_{i=1}^{n}\left[(2)\frac{3}{n}\right]$$

$$= \lim_{n\to\infty}\left[\left(1\cdot\frac{3}{n}\right)\frac{3}{n} + \left(2\cdot\frac{3}{n}\right)\frac{3}{n} + \cdots + \left(n\cdot\frac{3}{n}\right)\frac{3}{n}\right] + \lim_{n\to\infty}\left[\frac{6}{n}+\frac{6}{n}+\cdots+\frac{6}{n}\right]$$

$$= \lim_{n\to\infty}\left[(1+2+\cdots+n)\frac{3}{n}\frac{3}{n}\right]+6 = \lim_{n\to\infty}\left[\frac{(1+n)n}{2}\frac{3}{n}\frac{3}{n}\right]+6$$

$$= \lim_{n\to\infty}\left[\frac{(n+n^2)}{2}\frac{9}{n^2}\right]+6 = \lim_{n\to\infty}\left[\frac{9}{2}\left(\frac{1}{n}+1\right)\right]+6 = \frac{9}{2}+6 = \frac{21}{2}$$

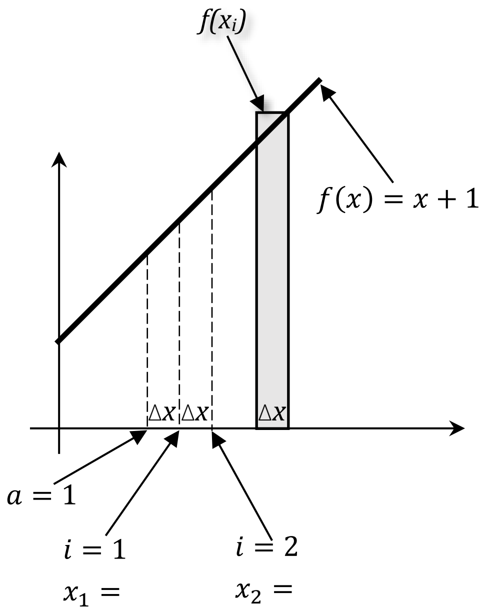
Exercise 3: Fill in the table to evaluate the definite integrals as limits

$$\int_{-1}^{5}(3x+10)\,dx \qquad \int_{0}^{2}\sqrt{x}\,dx \qquad \int_{1}^{3}\frac{1}{x}dx \qquad \int_{0}^{\pi/2}\sin x\,dx$$

a=	b=	$\Delta x =$	$x_1 =$	$x_2 =$	$x_i =$	$f(x_i) =$
-1	5	6/n	-1+1(6/n)	-1+2(6/n)	-1+i(6/n)	7+18i/n
0	2	2/n	0+1(2/n)	0+2(2/n)	0+i(2/n)	$\sqrt{2i/n}$
1	3	2/n	1+1(2/n)	1+2(2/n)	1+i(2/n)	n/(n+2i)
0	$\pi/2$	$\pi/2n$	$0+1[\frac{\pi}{2n}]$	$0+2[\frac{\pi}{2n}]$	$0+i[\frac{\pi}{2n}]$	$\sin\frac{i\pi}{2n}$

Exercise 1: Evaluate the following definite integrals as a limit

$$\int_1^3 f(x)\,dx = \int_1^3 (x+1)\,dx$$



Solution 将[a,b]区间分为 n 份，每份是一个小长方形。每一个小长方形:

$\Delta x = \dfrac{\mathrm{b}-\mathrm{a}}{\mathrm{n}} =$

$x_1 =$

$x_2 =$

$x_i =$

$f(x_i) =$

$=$

$$S = \int_1^3 (x+1)\,dx = \lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i)\Delta x_i$$

$=$

$=$

$=$

$=$

Exercise 2: Evaluate the limit

$$\lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

Over the region bounded by the graphs of the equations.

$$f(x) = x + 2, y = 0, x = 0, x = 3$$

Solution $\Delta x =$

$x_1 =$

$x_2 =$

$x_i =$

$f(x_i) =$

$S =$

Exercise 3: Fill in the table to evaluate the definite integrals as limits

$$\int_{-1}^{5}(3x+10)\,dx \qquad \int_{0}^{2}\sqrt{x}\,dx \qquad \int_{1}^{3}\frac{1}{x}\,dx \qquad \int_{0}^{\pi/2}\sin x\,dx$$

a=	b=	$\Delta x =$	$x_1 =$	$x_2 =$	$x_i =$	$f(x_i) =$

Exercise4: (p293) Evaluate the definite integral of the algebraic function.

9. $\int_{-1}^{1}(t^2-2)dt$	$=\left(\frac{t^3}{3}-2t\right)_{-1}^{1}$ $=\left(\frac{1}{3}-2\right)-\left(\frac{-1}{3}+2\right)$ $=\frac{-10}{3}$
10. $\int_{1}^{2}(6x^2-3x)dx$	$=\left(6\frac{x^3}{3}-3\frac{x^2}{2}\right)_{1}^{2}=\left(2x^3-3\frac{x^2}{2}\right)_{1}^{2}$ $=\left(2\cdot 2^3-3\frac{2^2}{2}\right)-\left(2-3\frac{1}{2}\right)$ $=\frac{19}{2}$
11. $\int_{0}^{1}(2t-1)^2dt$	$=\int_{0}^{1}(4t^2-4t+1)dt$ *Rewrite* $=\left(4\frac{t^3}{3}-4\frac{t^2}{2}+t\right)_{0}^{1}$ *Integrate* $=4\frac{1}{3}-4\frac{1}{2}+1=\frac{1}{3}$ *Simplify*
12. $\int_{-1}^{1}(t^3-9t)dt$	$=\left(\frac{t^4}{4}-9t^2\right)_{-1}^{1}$ $=\left(\frac{1}{4}-9\right)-\left(\frac{1}{4}-9\right)$ $=0$
13. $\int_{1}^{2}\left(\frac{3}{x^2}-1\right)dx$	$=\int_{1}^{2}(3x^{-2}-1)dx$ $=\left(3\frac{x^{-1}}{-1}-x\right)_{1}^{2}=\left(\frac{-3}{x}-x\right)_{1}^{2}$ $=\left(\frac{-3}{2}-2\right)-\left(\frac{-3}{1}-1\right)$ $=\frac{-3}{2}-2+3+1=\frac{1}{2}$
14. $\int_{-2}^{-1}\left(u-\frac{1}{u^2}\right)du$	$=\int_{-2}^{-1}(u-u^{-2})du$ $=\left(\frac{u^2}{2}-\frac{u^{-1}}{-1}\right)_{-2}^{-1}=\left(\frac{u^2}{2}+\frac{1}{u}\right)_{-2}^{-1}$ $=\left(\frac{1}{2}+\frac{1}{-1}\right)-\left(\frac{4}{2}+\frac{1}{-2}\right)$

	$=\frac{1}{2}-1-2+\frac{1}{2}=-2$
15. $\int_1^4\left(\frac{u-2}{\sqrt{u}}\right)du$	$=\int_1^4(u^{1/2}-2u^{-1/2})du$ $=\left(\frac{u^{\frac{3}{2}}}{3/2}-2\frac{u^{\frac{1}{2}}}{1/2}\right)_1^4=\left(\frac{2}{3}u^{\frac{3}{2}}-4u^{\frac{1}{2}}\right)_1^4$ $=\left(\frac{2}{3}4^{\frac{3}{2}}-4\cdot4^{\frac{1}{2}}\right)-\left(\frac{2}{3}-4\right)$ $=\left(\frac{2}{3}\cdot8-8\right)-\left(\frac{2}{3}-4\right)=\frac{2}{3}$
16. $\int_{-3}^3 v^{1/3}dv$ 17. $\int_{-1}^1(\sqrt[3]{t}-2)dt$	$=\left(\frac{v^{\frac{4}{3}}}{4/3}\right)_{-3}^3$ $=\frac{3}{4}\left(3^{\frac{4}{3}}\right)-\frac{3}{4}\left(-3^{\frac{4}{3}}\right)=0$ $=\int_{-1}^1(t^{1/3}-2)dt$ $=\left(\frac{t^{\frac{4}{3}}}{4/3}-2t\right)_{-1}^1$ $=\left(\frac{3}{4}-2\right)-\left(\frac{3}{4}+2\right)=-4$
18. $\int_1^8\sqrt{\frac{2}{x}}dx$	$=\sqrt{2}\int_1^8 x^{-1/2}dx$ $=\sqrt{2}\left(\frac{x^{\frac{1}{2}}}{1/2}\right)_1^8=2\sqrt{2}(\sqrt{x})_1^8$ $=2\sqrt{2}(\sqrt{8}-\sqrt{1})=8-2\sqrt{2}$

Exercise 5: (p293) evaluate the definite intergral of the trigonometric function.

27. $\int_0^\pi(1+\sin x)dx$	$=(x-\cos x)_0^\pi$ $=(\pi-\cos\pi)-(0-\cos 0)$ $=\pi+1-0+1$ $=2+\pi$
28. $\int_0^\pi(2+\cos x)dx$	$=(2x+\sin x)_0^\pi$ $=(2\pi+\sin\pi)-(0+\sin 0)$ $=2\pi+0-0-0$ $=2\pi$

29. $\int_0^{\pi/4} \frac{1-\sin^2\theta}{\cos^2\theta}\,d\theta$

$$= \int_0^{\pi/4} \frac{\cos^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\pi/4} d\theta$$

$$= (\theta)_0^{\pi/4} = \pi/4$$

30. $\int_0^{\pi/4} \frac{\sec^2\theta}{\tan^2\theta+1}\,d\theta$

$$= \int_0^{\pi/4} \frac{\sec^2\theta}{\sec^2\theta}\,d\theta = \int_0^{\pi/4} d\theta$$

$$= (\theta)_0^{\pi/4} = \frac{\pi}{4}$$

31. $\int_{-\pi/6}^{\pi/6} \sec^2 x\,dx$

$$= (\tan x)_{-\pi/6}^{\pi/6}$$

$$= (\sqrt{3}/3) - (-\sqrt{3}/3)$$

$$= \frac{2\sqrt{3}}{3}$$

32. $\int_{\pi/4}^{\pi/2} (2-\csc^2 x)dx$

$$= (2x+\cot x)_{\pi/4}^{\pi/2}$$

$$= \left[\pi + \cot\left(\frac{\pi}{2}\right)\right] - [\frac{\pi}{2} + \cot\left(\frac{\pi}{4}\right)]$$

$$= \pi + 0 - \frac{\pi}{2} - 1$$

$$= \frac{\pi}{2} - 1$$

33. $\int_{-\pi/3}^{\pi/3} 4\sec\theta\tan\theta\,d\theta$

$$= (4\sec\theta)_{-\pi/3}^{\pi/3} = \left(\frac{4}{\cos\theta}\right)_{-\pi/3}^{\pi/3}$$

$$= \frac{4}{\cos\frac{\pi}{3}} - \frac{4}{\cos\frac{-\pi}{3}}$$

$$= 4/(\frac{1}{2}) - 4/(\frac{1}{2}) = 0$$

34. $\int_{-\pi/2}^{\pi/2} (2t+\cos t)dt$

$$= \left(2\frac{t^2}{2} + \sin t\right)_{-\pi/2}^{\pi/2} = (t^2+\sin t)_{-\pi/2}^{\pi/2}$$

$$= \left[\left(\frac{\pi}{2}\right)^2 + \sin\frac{\pi}{2}\right] - [(\frac{-\pi}{2})^2 + \sin\frac{-\pi}{2}]$$

$$= \left(\frac{\pi}{2}\right)^2 + 1 - \left(\frac{\pi}{2}\right)^2 + 1 = 2$$

Exercise 5: (p293) find the value(s) of c guaranteed by the Mean Value Theorem for Integrals for the function over the given interval.

45. $f(x) = x^3$, $[0,3]$

$f(c)(b-a) = \int_0^3 x^3 dx$ *Rewrite*

$c^3(3-0) = \frac{x^4}{4}\Big|_0^3$ *Integrate*

$3c^3 = \frac{3^4}{4} \Rightarrow c^3 = \frac{3^3}{4} = \frac{27}{4}$ *Simplify*

$c = \frac{3}{\sqrt[3]{4}}$

46. $f(x) = \frac{9}{x^3}$, $[1,3]$	$f(c)(b-a) = \int_1^3 \frac{9}{x^3}dx = 9\int_1^3 x^{-3}dx$ $\frac{9}{c^3}(3-1) = 9\left(\frac{x^{-2}}{-2}\right)_1^3$ $\frac{2}{c^3} = \left(\frac{-1}{2x^2}\right)_1^3 = \frac{-1}{2(3^2)} - \frac{-1}{2(1^2)} = \frac{-1}{18} + \frac{1}{2} = \frac{8}{18}$ $c^3 = 2\left(\frac{18}{8}\right) = \frac{9}{2} \Rightarrow \mathrm{c} = \sqrt[3]{\frac{9}{2}}$
47. $f(x) = \sqrt{x}$, $[4,9]$	$f(c)(b-a) = \int_4^9 \sqrt{x}dx = \int_4^9 x^{1/2}dx$ $\sqrt{c}(9-4) = \left(\frac{x^{\frac{3}{2}}}{3/2}\right)_4^9 = \left(\frac{2}{3}x^{\frac{3}{2}}\right)_4^9$ $5\sqrt{c} = \frac{2}{3}\left(9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right) = \frac{2}{3}(27-8) = \frac{38}{3}$ $\mathrm{c} = \left(\frac{38}{15}\right)^2 = \frac{1444}{225}$
48. $f(x) = x^2/4$, $[0,6]$	$f(c)(b-a) = \int_0^6 \frac{x^2}{4}dx = \frac{1}{4}\int_0^6 x^2dx$ $\left(\frac{c^2}{4}\right)(6-0) = \frac{1}{4}\left(\frac{x^3}{3}\right)_0^6$ $6c^2 = 6^3/3 \Rightarrow c^2 = 6^2/3$ $\mathrm{c} = \pm 6/\sqrt{3} = \pm 2\sqrt{3}$ ($\mathrm{c} = -2\sqrt{3}$ is not on the interval) $\mathrm{c} = 2\sqrt{3}$
49. $f(x) = 2sec^2x$, $[-\frac{\pi}{4}, \frac{\pi}{4}]$	$f(c)(b-a) = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 2sec^2xdx$ $2sec^2c\left(\frac{\pi}{2}\right) = 2(\tan x)_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$ $\pi sec^2c = 2(1+1) = 4$ $\sec c = \sqrt{4/\pi}$ $\quad$ ($-\sqrt{4/\pi}$ is not *on* the interval) $\mathrm{c} = \pm\mathrm{arc\,sec}(\sqrt{4/\pi})$ *or* $\mathrm{c} = \pm\arccos(\frac{\sqrt{\pi}}{2})$
50. $f(x) = \cos x$, $[-\frac{\pi}{3}, \frac{\pi}{3}]$	$f(c)(b-a) = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos x\, dx$ $\cos c\left(\frac{2\pi}{3}\right) = (\sin x)_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$ $\cos c\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} - \left(\frac{-\sqrt{3}}{2}\right) = \sqrt{3}$ $\cos c = \frac{3\sqrt{3}}{2\pi}$

	$c = \pm \arccos(\frac{3\sqrt{3}}{2\pi})$

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